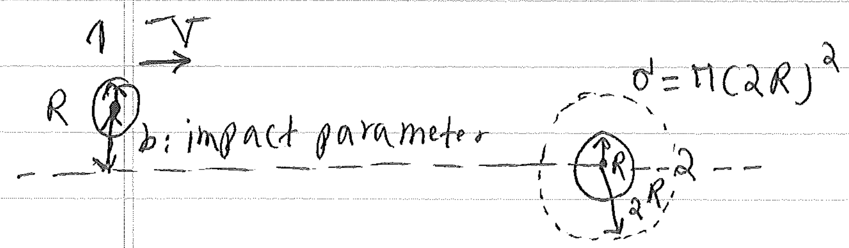
Lec 11: 10/02/2013

Thermal History of the Universe (Cont'd):

So far, we have assumed thermal equilibrium in the early universe in deriving the expressions for the number density and energy density of different species. Interactions among particles are essential in estabishing and maintaining the equilibrium. Therefore, we need to know the efficiency of particle interactions in order to find whether and when thermal equilibrium of a given species is achieved or lost.

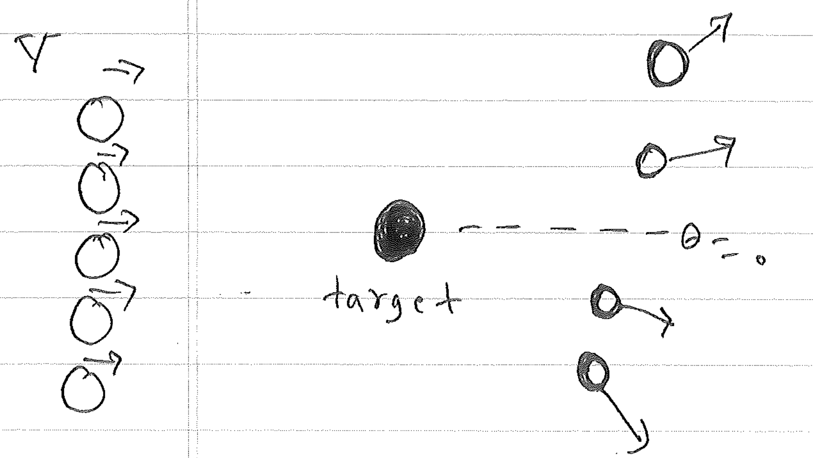
Let us consider two particles moving toward each other. Interaction between the particles results in momentum and/or energy exchange. A way to figure out whether the particles interacted with each other is to look at the direction of outgoing particles. A momentum exchange would change the direction of a particle and hence resulting in its deflection from the initial path.

As a simple, but instructive, example consider interaction of two billiard balls. Ball 1 is shot toward ball 2 that is at rest and considered to be target. Both balls are spheres with radius R. The top view of this experiment is as follows:



In this example, interaction between the balls occurs when they collide. It is clear that a collision happens if the vertical distance between the center of the two balls (called impact parameter "b") is less than or equal to $2R$. One can define a cross-sectional area $\sigma = \pi(2R)^2$ about the center of the target. Collision happens only if the center of ball 1 crosses this area. This is the cross section for collisions (in general the cross section for an interaction).

This cross section can also be measured experimentally. If one shoots a number of balls to the target, and measures the number of balls that come out deflected ($\theta \neq 0$), then σ can be found from the following relation:



$$\sigma = \frac{\text{Number of deflected balls per unit time}}{\text{Number of incident balls per unit area per unit time}}$$

In general, there are many targets in a given environment. In such a case, an interesting quantity to find is the rate at which a particle interacts with the targets in the medium, i.e., the number of interaction that the particle undergoes per unit time. The effective volume for interaction swept by the particle within time δt is $\sigma v_{rel} \delta t$, where v_{rel} is the relative velocity between the particle and targets. Interaction occurs if this volume contains one target, which is found to be $\sigma v_{rel} \delta t = \frac{1}{n}$.

Therefore, an interaction takes place every δt, which results in an interaction rate:

$$\Gamma = \frac{1}{\delta t} = n \sigma v_{rel} \qquad (I)$$

One can also define a mean free path, l, which is the distance that particle travels (freely) between two successive interactions. It is given by:

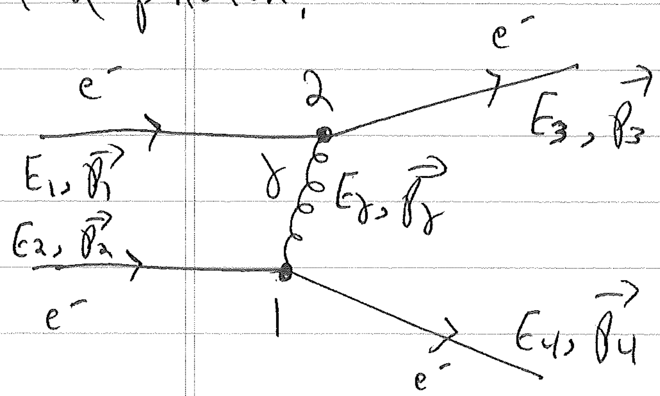
$$l = v_{rel} \delta t = \frac{1}{n\sigma}$$

In a general case, the relative velocity between the particle and all targets are not equal. One rather has a range of v_{rel} that are given by a distribution, for example, a thermal distribution. In this case, one should average over $n \sigma v_{rel}$ to find the rate:

$$\Gamma = \langle n\sigma v_{rel} \rangle \qquad (II)$$

In the case of elementary particles, interactions happen by the exchange of other elementary particles. This makes the situation very different from the collision of billiard balls. Elementary particles do not have a definite size and are not hard spheres. Therefore, the cross section for interaction does not have a strict geometrical interpretation. Perhaps a better example will be that of two skaters that interact via throwing a basketball. Depending on the mass and the velocity of the ball, skaters can interact at various distances from each other. Once all these possibilities are summed over properly, one can find a cross section. In the case of elementary particles, the relevant interactions are given by the underlying theory (i.e., the standard model). A detailed calculation of the interaction cross sections is beyond the scope of this course. However, it is possible to estimate some cross sections with heuristic arguments. We present such estimates for the electromagnetic and weak interactions.

1) Electromagnetic interactions: as an example, we consider interaction between two electrons that occurs by the exchange of a photon:



The interaction occurs via emission of a photon at point 1 and its absorption at point 2, where points 1 and 2 can be anywhere in the space. Conservation of energy and momentum in points 1 and 2 implies:

$E_\gamma = E_2 - E_4 = E_3 - E_1 \qquad \vec{p}_\gamma = \vec{p}_2 - \vec{p}_4 = \vec{p}_3 - \vec{p}_1$

For simplicity, we consider the relativistic regime where the energy of electrons is much larger than the electron mass m_e.

Now let us calculate $E_\gamma^2 - |\vec{p}_\gamma|^2$:

$$E_\gamma^2 - |\vec{P}_\gamma|^2 = (E_2 - E_4)^2 - |\vec{P}_2 - \vec{P}_4|^2 = E_2^2 + E_4^2 - 2E_2E_4 -$$

$$|\vec{P}_2|^2 - |\vec{P}_4|^2 + 2\vec{P}_2 \cdot \vec{P}_4 = (E_2^2 - |\vec{P}_2|^2) + (E_4^2 - |\vec{P}_4|^2) -$$

$$2(E_2E_4 - \vec{P}_2 \cdot \vec{P}_4) \simeq 2m_e^2 - 2E_2E_4(1-\cos\theta) \simeq -2E_2E_4(1-\cos\theta)$$

Here θ is the angle between $\vec{P}_2$ and $\vec{P}_4$. Since $-1 \le \cos\theta \le 1$, we have:

$$E_\gamma^2 - |\vec{P}_\gamma|^2 \sim -2E_2E_4$$

For a free photon with energy E_γ and momentum $\vec{P}_\gamma$ we have $E_\gamma^2 - |\vec{P}_\gamma|^2 = 0$ from special relativity. The fact that this does not hold for the exchanged photon means that this photon is more like a resonance that has a finite lifetime:

$$\delta t \sim |E_\gamma^2 - |\vec{P}_\gamma|^2|^{-\frac{1}{2}} \sim \frac{1}{(E_1E_2)^{\frac{1}{2}}}$$

Being a relativistic particle, one can estimate the range for the exchange of such a photon:

$$\delta r \sim \frac{e^2}{(E_1E_2)^{\frac{1}{2}}}$$

The factor e^2 in the numerator appears because the emission and absorption of the exchanged photon each introduce a factor e (the charge of electron). From this, the cross section for interaction of two electrons can be estimated as:

$$\sigma \sim \frac{e^4}{E_2 E_4} \sim \frac{\alpha_{em}^2}{E_2 E_4} \qquad \alpha_{em} \equiv \frac{e^2}{4\pi} \left(= \frac{1}{137} \text{ at low energies} \right)$$

Here we have ignored prefactors in the relation. The rate for a typical electromagnetic interaction after thermal averaging is then estimated to be:

$$\Gamma_{em} \sim \frac{\alpha^2}{T^2} T^3 \sim \alpha^2 T \quad (\text{III})$$

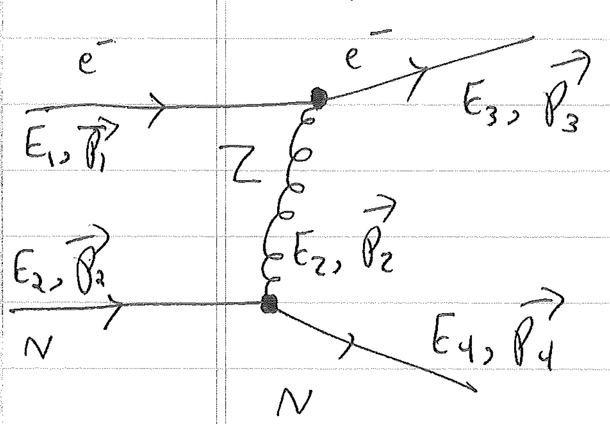
Here we have used $n \propto T^3$ and $E \propto T$, as is the case in a thermal ensemble. Also, in the relativistic regime $v_{rel} \sim 1$.

In order for electromagnetic interaction be efficient in the early universe, we must have $\Gamma_{em} > H$. Since $H \sim \frac{3T^2}{M_P}$ at high temperatures, this leads to:

$$\Gamma_{em} > H \Rightarrow \alpha^2 T > \frac{3T^2}{M_P} \Rightarrow T \lesssim 10^{15} \text{ GeV}$$

Therefore, at temperatures below $\sim 10^{15}$ GeV, electromagnetic interactions are efficient and can keep thermal equilibrium of charged particles and photons.

2) Weak interactions: as an example, we consider interactions of neutrinos with electrons. This happens via the exchange of a Z particle as follows:



$m_Z \sim 90$ GeV

At very high temperatures, $T \gg 100$ GeV, the mass of Z does not have a significant effect. In this case, the rate for $e^- \nu$ interactions via Z exchange will be similar to that of the electrons given in Eq. (3). We therefore focus on the case where $T \ll m_Z$ (but still $T \gg m_e$).

In this regime, we can find $E_Z^2 - |\vec{P}_Z|^2$:

$$E_Z^2 - |\vec{P}_Z|^2 \sim -2 E_2 E_4$$

This results in a lifetime for the exchange Z as follows:

$$\delta t \sim \left| E_Z^2 - |\vec{P}_Z|^2 - m_Z^2 \right|^{-\frac{1}{2}} \sim \frac{1}{m_Z}$$

The range of the exchanged Z is found to be:

$$\delta r \sim v \delta t \sim \frac{|\vec{P}_Z|}{m_Z} \delta t \sim \frac{E_2 - E_4}{m_Z^2}$$

The cross section is given by:

$$\sigma \sim \frac{\alpha_W^2 E_2^2}{m_Z^4}$$ (α_W: weak interaction fine structure constant)

In a thermal ensemble, one finds the following estimate for the rate of weak interactions:

$$\Gamma_W \sim \frac{\alpha_W^2 T^5}{m_Z^4} \sim G_F^2 T^5 \quad (\mathrm{IV})$$

Here $G_F = 1.17 \times 10^{-5}$ GeV^{-2} is the Fermi constant. The condition for weak interactions to be efficient will then be:

$$\Gamma_W > H \Rightarrow G_F^2 T^5 > \frac{T^2}{M_P} \Rightarrow T^3 > \frac{1}{G_F^2 M_P}$$

Pluging the numbers in, we find:

$T \gtrsim O(\mathrm{MeV})$

Therefore, weak interactions are efficient when temperature of the thermal bath is above MeV. At lower temperatures, they cannot keep up with the expansion of the universe, hence become ineffective.

This has very important consequences for the neutrinos since these particles have only weak interactions. Moreover, processes that are governed by the weak interactions are also affected when $T < 1$ MeV. We will discuss these issues in more detail in the coming lectures.